DIFFERENTIAL EQUATION

Some questions (Assertion–Reason type) are given below. Each question contains **Statement – 1 (Assertion)** and **Statement – 2 (Reason)**. Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct. So select the correct choice :

Choices are :

(A) **Statement – 1** is True, **Statement – 2** is True; **Statement – 2 is** a correct explanation for **Statement – 1**.
(B) **Statement – 1** is True, **Statement – 2** is True; **Statement – 2 is NOT** a correct explanation for **Statement – 1**.
(C) **Statement – 1** is True, **Statement – 2** is False.
(D) Statement – 1 is False, Statement – 2 is True.

227. **Statement-1:** The order of the differential equation whose general solution is $y = c_1\cos 2x + \cos_2\sin^2 x + c_3\cos^2 x + c_4e^{2x} + c_5\ e^{2x+c_6}$ is 3
Statement-2: Total number of arbitrary parameters in the given general solution in the statement (1) is 6.

228. **Statement-1:** Degree of differential equation of parabolas having their axis along x–axis and vertex at (2, 0) is 2.
Statement-2: Degree of differential equation of parabola having their axis along x–axis and vertex at (1, 0) is 1.

229. **Statement–1 :** Solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = x$ is $xy = \frac{x^3}{3} + c$.
Statement–2 : Solution of the differential equation $\frac{dy}{dx} + PY = Q$ is
$Ye^{\int pdx} = \int\left(Q.e^{\int pdx}\right)dx + c$ where P and Q are function of x alone.

230. Let the general solution of a differential equation be $y = ae^{bx+c}$.
Statement–1 : Order of the differential equation is 3.
Statement–2 : Order of the differential equation is equal to the number of actual constant of the solution

231. Let F be the family of ellipses on the Cartesian plane, whose directrices are $x = \pm 2$.
Statement–1 : The order of the differential equation of the family F is 2.
Statement–2 : F is a two parameter family.

232. Consider the differential equation $(x^2 + 1).\frac{d^2y}{dx^2} = 2x.\frac{dy}{dx}$.
Statement–1 : For any member of this family $y \to \infty$ as $x \to \infty$.
Statement–2 : Any solution of this differential equation is a polynomial of odd degree with positive coefficient of maximum power.

233. **Statement–1 :** The solution of the differential equation $x\frac{dy}{dx} = y(\log y - \log x + i)$ is $y = xe^{cx}$.
Statement–2 : A solution of the differential equation $\left(\frac{dy}{dx}\right)^2 - x\left(\frac{dy}{dx}\right) + y = 0$ is $y = 2$.

234. **Statement-1:** Order of the differential equation of family of parabola whose axis is perpendicular to y–axis and ratus rectum is fix is 2.
Statement-2: Order of first equation is same as actual no. of abitrary constant present in diff. equation.

235. **Statement-1:** Solution of $y\,dy = x - x$ as is family of rectangular hyperbola
Statement-2: Solution of $y\frac{dy}{dx} = 1$ is family of parabola

236. **Statement-1:** Solution of differential equation $dy\,(x^2y - 1) + dx\,(y^2x - 1) = 0$ is $\frac{x^2y^2}{2} = x + y + c$
Statement-2: Order of differential equation of family of circle touching the coordinate axis is 1.

237. **Statement-1:** Integrating factor of $\frac{dy}{dx} + y = x^2$ is e^x

Statement-2: Integrating factor of $\frac{dy}{dx}+p(x)y=Q(x)$ is $e^{\int p(x)dx}$

238. **Statement-1:** The differential equation of all circles in a plane must be of order 3.
Statement-2: There is only one circle passing through three non-collinear points.

239. **Statement-1:** The degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^{2/3}+6-2\frac{d^2y}{dx^2}+15\frac{dy}{dx}=0$ is 3.

Statement-2: The degree of the highest order derivative occuring in the D.E. when the D.E. has been expressed as a polynomial of derivatives.

240. **Statement-1:** Solution of $\frac{x+y\frac{dy}{dx}}{y-x\frac{dy}{dx}}=\frac{x\cos^2(x^2+y^2)}{y^3}$ is $\frac{x^2}{y^2}$ - tan (x^2+y^2) = c

Statement-2: Since the given differential equation is homogenous can be solved by putting y = vx

241. **Statement-1:** The order of the differential equation formed by the family of curve $y = c_1e^x + (c_2 + c_3)\, e^{x+c_4}$ is '1'. Here c_1, c_2, c_3, c_4 are arbitrary constant.
Statement-2: The order of the differential equation formed by any family of curve is equal to the number of arbitrary constants present in it.

242. **Statement-1:** The degree of differential equation $3\sqrt{1+\left(\frac{dy}{dx}\right)^2}=\log\left(\frac{d^2y}{dx^2}\right)$ is not defined.

Statement-2: The degree of differential equation is the power of highest order derivative when differential equation has been expressed as polynomial of derivatives.

243. **Statement-1:** The order of differential equation of family of circles passing then origin is 2.
Statement-2: The order of differential equation of a family of curve is the number of independent parameters present in the equation of family of curves

244. **Statement-1:** Integrating factor of $\frac{xdy}{dx}+3y=x$ is x^3

Statement-2: Integrating factor of $\frac{dy}{dx}+p(x)y=Q(x)$ is $e^{\int pdx}$

245. **Statement-1:** The differentiable equation $y^3dy + (x + y^2)\, dx = 0$ becomes homogeneous if we put $y^2 = t$.
Statement-2: All differential equation of first order and first degree becomes homogeneous if we put y = tx.

246. **Statement-1:** The general solution of $\frac{dy}{dx}$ + P(x) y = Q(x) is $e^{\int p(x)dx}+c$

Statement-2: Integrating factor of $\frac{dy}{dx}$ + P(x) y = Q(x) is $e^{\int p(x)dx}$

247. **Statement-1:** The general solution of $\frac{dy}{dx}+y=1$ is $ye^x = e^x + c$

Statement-2: The number of arbitrary constants in the general solution of the differential equation is equal to the order of differential equation.

248. **Statement-1:** Degree of the differential equation $y=x\times\frac{dy}{dx}+\sqrt{1+\left(\frac{dy}{dx}\right)^2}$ is 2.

Statement-2: In the given equation the power of highest order derivative when expressed as a polynomials in derivatives is 2.

249. **Statement-1:** The differential equation of the family of curves represented by $y = A.e^x$ is given by $\frac{dy}{dx} = y$.

Statement-2: $\frac{dy}{dx} = y$ is valid for every member of the given family.

250. **Statement-1:** The differential equation $\frac{dy}{dx} = \frac{2xy}{x^2+y^2}$ can be solved by putting $y = vx$

Statement-2: Since the given differentiable equation is homogenous

251. **Statement-1:** A differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ can be solved by finding. If = $e^{\int Pdx}$

$= e^{\int 1/x dx} = e^{\log x} = x$ then solution $y.x = \int x^3 dx + c$

Statement-2: Since the given differential equation in of the form $dy/dx + py = \phi$ where p, ϕ are function of x

252. **Statement-1:** The differential equation of all circles in a plane must be of order 3.
Statement-2: There is only on circle passing through three non collinear points.

ANSWER

227. A		**228. D**		**229. A**	**230. D**		**231. A**	
232. A	**233. C**	**234. A**		**235. D**	**236. B**		**237. A**	**238. A**
239. D	**240. C**	**241. C**	**242. A**	**243. A**		**244. A**		**245. C**
246. D	**247. B**	**248. A**	**249. A**		**250. A**	**251. A**	**252. A**	

DETAILS SOLUTION

227. $y = c_1 \cos 2x + c_2 \sin^2 x + c_3 \cos^2 x + c_4 e^{2x} + c_5\, e^{2x+c_6}$

$= c_1 \cos 2x + c_2 \left[\frac{1-\cos 2x}{2}\right] + c_3\left[\frac{\cos 2x - 1}{2}\right] + c_4 e^{2x} + c_5 e^{2x}.e^{c_6}$

$= \left(c_1 - \frac{c_2}{2} + \frac{c_3}{2}\right)\cos 2x + \left(\frac{c_2}{2} - \frac{c_3}{2}\right) + (c_4 + c_5')e^{2x} = \lambda_1 \cos 2x + \lambda_2\, e^{2x} + \lambda_3$

$\Rightarrow$ Total number of independent parameters in the given general solution is 3. Ans. : **A**

228. Equation of parabola will be $y^2 = ap\,(x-1)$

$\Rightarrow 2y\,\frac{dy}{dx} = p \Rightarrow$ D.E. is $y = 2\,\frac{dy}{dx}(x-1) \Rightarrow$ degree of this D.E. is 1. **Ans. : D**

229. (a)

$e^{\int Pdx} = e^{\int \frac{dx}{x}} = x$

$\therefore$ Sol. is $xy = \int x^2 dx + c$

$xy = \frac{x^3}{3} + c$.

230. (D)
$y = ae^{bx+c} = ae^c.\, e^{bx} = Ae^{bx}$
$\therefore$ order is two.

231. Statement – II is true as any member of the family will have equation $\frac{x^2}{a^2}+\frac{(y-\beta)^2}{a^2(1-e^2)}=1$, where 0 < e < 1, a > 0, b ∈ R and ae = 2.
Hence F is a two parameter family.
Statement – I is true, because of statement – II, because order of a differential equation of a n parameter family is n.
Hence (a) is the correct answer.

232. The given differential equation is $\frac{d\left(\frac{dy}{dx}\right)}{\frac{dy}{dx}}=\frac{2x}{x^2+1}dx$

$\Rightarrow \ell n\left(\frac{dy}{dx}\right)=\ell n\left(x^2+1\right)+\ell nc,\ c>0 \Rightarrow \frac{dy}{dx}=c\left(x^2+1\right) \Rightarrow y=c\left(\frac{x^3}{3}+x\right)+c'$, $c' \in R$.
Obviously $y \to \infty$, as $x \to \infty$; as $c > 0$
Hence (a) is the correct answer.

233. The given equation can be rearranged as,
$\frac{dy}{dx}=\frac{y}{x}\left(\log\left(\frac{ye}{x}\right)\right)$

put $y = vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$ $\Rightarrow \frac{dv}{dx}=\frac{v\log v}{x} \Rightarrow \int\frac{dv}{v\log v}=\int\frac{dx}{x} \Rightarrow y = xe^{cx}$

for II, put $\frac{dy}{dx}=p \Rightarrow p^2-xp+y=0$

$\Rightarrow y= px-p^2 \Rightarrow p=p+x\frac{dp}{dx}-2p\frac{dp}{dx} \Rightarrow \frac{dp}{dx}=0$ or $x-2p=0 \Rightarrow y=2x+c$

Hence (c) is the correct answer.

234. $(x-h)^2=4b(y-k)$
here b is constant and h, k are parameters
Hence order is 2. Ans. : A

235. (D) $\int ydy=\int dx-\int dx$
$\frac{y^2}{2}+\frac{x^2}{2}=x+c$ is family of circle $\int ydy=\int dx \Rightarrow \frac{y^2}{2}=x+c$ which is family of parabola

236. $\int xy\, d(xy)=\int d(x+y)$
$\frac{x^2y^2}{2}=x+y+c$
let circle is $(x-h)^2+(y-h)^2=h^2$
Hence order of differential equation will be 1. Ans. : B

237. Option (a) is correct. I.F. = $e^{\int f.dx}=e^x$

238. Option (a) is correct
The equation of circle contains. Three independent constants if it passes through three non-collinear points, therefore a is true and follows from R.

239. $\left(\frac{d^3y}{dx^3}\right)^3=\left(2\frac{d^2y}{dx^2}-15\frac{dy}{dx}-6\right)^2$

Hence degree is 2. Ans. (D)

240. $\dfrac{2x\,dx+2y\,dy}{\cos^2(x^2+y^2)}=\dfrac{2x}{y}\left(\dfrac{ydx-xdy}{y^2}\right)$

$\Rightarrow \int \sec^2(x^2+y^2)\,(2x\,dx+2y\,dy) = 2\int \dfrac{x}{y}.d\left(\dfrac{x}{y}\right)$

$\Rightarrow \tan(x^2+y^2) = \dfrac{2.(x^2/y^2)}{2}+c$

$\Rightarrow \dfrac{x^2}{y^2}-\tan(x^2+y^2)=c$ Ans. (C)

241. $y = c_1e^x + (c_2 + c_3)\,e^x \times e^{c_4} = e^x\,(c_1 + (c_2 + c_3)\ e^{c_4})$

$y = ce^x$ … (1) $\{\text{here } c=c_1+(c_2+c_3)e^{c_4}\}$

$\dfrac{dy}{dx}=ce^x$

$c=\dfrac{\frac{dy}{dx}}{e^x}$ Put in (1) $y=\dfrac{\frac{dy}{dx}}{e^x}\times e^x$

So $\dfrac{dy}{dx}=y$ and order is 1. 'c' is correct.

242. $\sqrt[3]{1+\left(\dfrac{dy}{dx}\right)^2}=\log\left(\dfrac{d^2y}{dx^2}\right)$

$1+\left(\dfrac{dy}{dx}\right)^2=\left(\log\left(\dfrac{d^2y}{dx^2}\right)\right)^3$

degree is not defined as it is not a polynomial of derivatives.
'a' is correct.

244. I.F. $e^{\int pdx} = e^{3\int \frac{1}{x}dx}$

$\dfrac{dy}{dx}+\dfrac{3y}{x}=1 = x^3.$

245. (C)

R is false since $\dfrac{dy}{dx}=\dfrac{x+y^2}{y+x^2}$ cannot be made homogenous by putting $y = tx$.

But if we put $y^2 = t$ in the differential equation in assertion A then $2y\dfrac{dy}{dx}=\dfrac{dt}{dx}$

And differential equation becomes $t\,.\,\dfrac{1}{2}\,dt + (x + t)\,dx = 0$

or $dx/dt - \dfrac{-t}{2(x+t)}$ which is homogeneous.

246. (D)
Statement-1 is false
Statement-2 is true.

247. (b) $\frac{dy}{dx} + y = 1 \Rightarrow \frac{dy}{1-y} = dx$

$\int \frac{dy}{1-y} = \int dx - \log(1-y) = x$

$1 - y = e^{-x}$, $ye^{x} = e^{x} + c$

order of differential equation is the number of arbitrary constants.

Both one true, but Statement-2 is not the correct explanation.

248. (A)

$$y = x\frac{dy}{dx} + \sqrt{1+\left(\frac{dy}{dx}\right)^2} \text{ becomes}$$

$$(x^2-1)\left(\frac{dy}{dx}\right)^2 - 2xy\frac{dy}{dx} + (y^2-1) = 0,$$ when expressed as a polynomial in derivatives.

249. (A)

$y = A.e^{x}$

on differentiation we get $\frac{dy}{dx} = A.e^{x}$

250. $\frac{dy}{dx} = \frac{2xy}{x^2+y^2}$ ... (1)

This is homogenous differential equation put $y = vx$

from (1) $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + \frac{xdv}{dx} = \frac{2x^2v}{x^2(1+v^2)}$

$x\frac{dv}{dx} = \frac{2v}{1+v^2} - v = \frac{2v - v - v^3}{1+v^2} = \frac{v(1-v^2)}{1+v^2}$

$\int \frac{(1+v^2)}{v(1-v^2)}dv = \int \frac{dx}{x}$

251. $dy/dx + y/x = x^2$ ... (1)

This is term of linear differential equation $dy/dx + py = \phi$ ... (2)

from (1) and (2) $p = -1/x$, $\phi = x^2$

I.f. $e^{\int Pdx} = e^{\int 1/x dx = x} e$

$y.I.f = \int x \times I.fd + c$

$yx = \int x^3 dx + c.$

Ans. (A)

252. (A)

The equation of circle contains three independent constants if it passes through three non-collinear points therefore A is true and follows from statement-2